

# Experimental realization of a generalized measuring device via a one-dimensional photonic quantum walk

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We demonstrate an implementation of unambiguous state discrimination of two equally probable single-qubit states via a one-dimensional photonic quantum walk. Furthermore we experimentally realize a quantum walk algorithm for implementing a generalized measurement in terms of positive operator value measurement on a single qubit. The measurement of the single-photons' positions corresponds to a measurement of an element of the positive operator value measurement on the polarizations of the single-photons.

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Quantum walks (QWs) exhibit distinct features compared to classical random walks (RWs) [1] and hence can be used to develop quantum algorithms [2–5] and to serve as an ideal test-bed for studying quantum effects such as Anderson localization [6–15], quantum chaos [16–20] and energy transport in photosynthesis [21, 22]. Furthermore, discrete-time QW is a process in which the evolution of a quantum particle on a lattice depends on a state of a coin. The coin degrees of freedom offer the potential for a wider range of controls over the evolution of the walker than are available in the continuous-time QW. Thus there are more applications for discrete-time QW such as a versatile platform for the exploration of a wide range of nontrivial topological effects [23–25] which have been implemented both theoretically and experimentally. Also QWs can be considered as a generalized measurement proposed theoretically in [26]. In such a scenario the measurement of the quantum walker at a certain position  $x = i$  corresponds to a measurement of an element  $E_i$  of a positive operator value measure (POVM) on a coin state.

Quantum mechanics forbids deterministic discrimination among nonorthogonal states. Nonetheless, the capability to distinguish nonorthogonal states unambiguously is an important primitive in quantum information processing. Unambiguous state discrimination between  $N$  states has  $N + 1$  outcomes: the  $N$  possible conclusive results, and the inconclusive result. Since no projective measurement in an  $N$ -dimensional Hilbert space can have more than  $N$  outcomes, generalized measurements such as POVMs are required. POVMs can be implemented by embedding the system into a larger Hilbert space and unitarily entangling it with the extra degrees of freedom (ancilla). Projective measurement of the ancilla induces an effective non-unitary transformation of the original system. By an appropriate design of the entangling unitary,

this effective non-unitary transformation can turn an initially nonorthogonal set of states into a set of orthogonal states with a finite probability of success. Based on this, the coupling between the walker and coin can be used to implement the generalized measurement. The coin is regarded as the target system whose states need to be discriminated. The walker is regarded as the ancilla and the outcomes of the projective measurement on it gives conclusive and inconclusive results of the state discrimination. The POVM element is obtained by taking the overlap between the final state of a properly engineered QW and the initial state of the walker, and then tracing the walker state out.

Unambiguous discrimination between two pure states has been demonstrated in optical systems [27–31]. In this work, we realize an unambiguous state discrimination of two equally probable single-qubit states via a one-dimensional discrete-time QW, and implement a method suggested in [26]. The QW indeed generates the POVM elements corresponding to the unambiguous state discrimination problem.

The motivation of the paper is to use a very simple experiment to show the meaning of the idea on using QW to implement a generalized measurement which is shown in [26]. A QW is based on projection measurement of the coin state, which can be extended in the evolution time. The projection measurement model can naturally be extended to an arbitrary generalized measurement. Compared to the standard approach to POVM in which projection measurements are performed on an extended Hilbert space, the extension of the Hilbert space is not needed for the QW scenario.

First we explain how we realize an unambiguous state discrimination of two pure single-qubit states through a QW optical-interferometer network. The goal is to find one of the two single-qubit states  $\{|0\rangle, \alpha|0\rangle + \beta|1\rangle\}$  with equal a priori probabilities from three outcomes: two possible conclusive results, and one inconclusive result.

The two states can always be represented by the two

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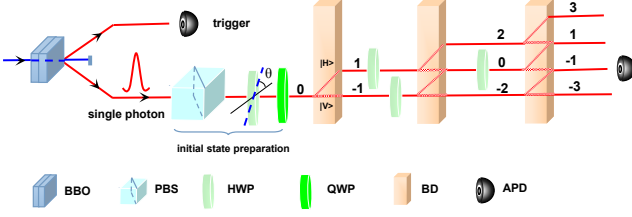


FIG. 1: (Color online.) Experimental schematic. Detailed sketch of the setup for realization of unambiguous state discrimination of two equally probable single-qubit states via a three-step QW. Single-photons are created via SPDC in two BBO crystals. One photon in the pair is detected to herald the other photon, which is injected into the optical network. Arbitrary initial coin states are prepared by a PBS, HWP and QWP. Position-dependent coin flipping is realized by the HWP with different setting angle placed in different optical path. Coincident detection of trigger and heralded photons at APDs yields data for the QW.

orthogonal states as

$$|\psi_{\pm}\rangle = \cos \frac{\phi}{2} |H\rangle \pm \sin \frac{\phi}{2} |V\rangle, \quad (1)$$

where  $\phi \in [0, \pi/2]$ ,  $|H\rangle$  ( $|V\rangle$ ) is the polarization state of a single photon: horizontal (vertical) and the condition

$$\cos \phi = \alpha \quad (2)$$

is satisfied.

The polarization degenerate photon pairs generated via type-I spontaneous parametric down-conversion (SPDC) in two 0.5mm-thick nonlinear- $\beta$ -barium-borate (BBO) crystals cut at  $29.41^\circ$ , pumped by a 400.8nm CW diode laser with up to 100mW of power. For 1D QWs, triggering on one photon prepares the other photon pairs at wavelength 801.6nm into a single-photon state  $|\psi_{\pm}\rangle$  via a polarizing beam splitter (PBS) following by waveplates. Interference filters determine the photon bandwidth 3nm and then pairs of down-converted photons are steered into the different optical modes (up and down) of the linear-optical network formed by a series of birefringent calcite beam displacers (BDs) and wave plates. Output photons are detected using avalanche photo-diodes (APDs, 7ns time window) with dark count rate of  $< 100\text{s}^{-1}$  whose coincident signals—monitored using a commercially available counting logic—are used to post-select two single-photon events. The total coincident counts are about  $1000\text{s}^{-1}$  (the coincident counts are collected over 40s). The probability of creating more than one photon pair is less than  $10^{-4}$  and can be neglected.

A standard model of a 1D discrete-time QW consists of a walker carrying a coin which is flipped before each step. In the basis of the coin state  $\{|H\rangle, |V\rangle\}$ , the position-dependent coin flipping operation for the  $n$ th step is given

by

$$C(x, n, \vartheta_{x,n}) = |x\rangle \langle x| \otimes \begin{pmatrix} \cos 2\vartheta_{x,n} & \sin 2\vartheta_{x,n} \\ \sin 2\vartheta_{x,n} & -\cos 2\vartheta_{x,n} \end{pmatrix} \quad (3)$$

with  $\vartheta_{x,n} \in [0, \pi/4]$ , consisting of a polarization rotation, which is realized with a half-wave plate (HWP) placed in the spatial modes  $x$  of the interferometer. The value of  $\vartheta$  is determined by the angle between the optic axis of HWP and horizontal. For some specific evolution, one can also use identity operator  $\mathbb{1}_x = |x\rangle \langle x| \otimes \mathbb{1}_c$  for a coin flipping.

The walker's positions are represented by longitudinal spatial modes. The conditional position shift due to the outcome of the coin flipping for each step

$$T = \sum_x |x+1\rangle \langle x| \otimes |H\rangle \langle H| + |x-1\rangle \langle x| \otimes |V\rangle \langle V| \quad (4)$$

acts on these modes, which is implemented by a BD with length 28mm and clear aperture  $10\text{mm} \times 10\text{mm}$ . The optical axis of each BD is cut so that vertically polarized photons are directly transmitted and horizontal photons moves up a 2.7mm lateral displacement into a neighboring mode which interferes with the vertical light in the same mode. Each pair of BDs forms an interferometer. BDs are placed in sequence and need to have their optical axes mutually aligned. Co-alignment ensures that beams split by one BD in the sequence yield maximum interference visibility after passing through a HWP and the next BD in the sequence. In our experiment, we attain interference visibility of 0.998 for each step. The unitary operation for the  $n$ th step is  $U_n = T [\sum_x C(x, n, \vartheta_{x,n}) + \sum_{x'} \mathbb{1}_{x'}]$ .

Back to the unambiguous state discrimination, it can be realized with a three-step QW. First the initial coin state  $|\psi(0)\rangle_c$  is initially prepared in  $|\psi_{\pm}\rangle$  and the walker starts from the original position  $x = 0$ . For the first step, the coin operation is identity and the unitary operation is  $U_1 = T$ . After going through the first BD, the photons are injected into the spatial mode  $\pm 1$  and the state of the walker+coin is

$$|\psi_{\pm}(1)\rangle = \cos \frac{\phi}{2} |1\rangle |H\rangle \pm \sin \frac{\phi}{2} |-1\rangle |V\rangle. \quad (5)$$

The unitary operation for the second step is

$$U_2 = T \left[ \sum_{x \neq \pm 1} |x\rangle \langle x| \otimes \mathbb{1}_c + C(-1, 2, \frac{\pi}{4}) + C \left( 1, 2, \frac{1}{2} \arccos \sqrt{1 - \tan^2 \frac{\phi}{2}} \right) \right], \quad (6)$$

and the state of the walker+coin is

$$|\psi_{\pm}(2)\rangle = \sqrt{\cos \phi} |2\rangle |H\rangle + \sin \frac{\phi}{2} |0\rangle (|V\rangle \pm |H\rangle). \quad (7)$$

Finally after the transformation for the third step

$$U_3 = T \left[ C(0, 3, \frac{\pi}{8}) + \sum_{x \neq 0} |x\rangle \langle x| \otimes \mathbb{1}_c \right] \quad (8)$$

the final state is

$$\begin{aligned} |\psi_+(3)\rangle &= \sqrt{\cos \phi} |3\rangle |H\rangle + \sqrt{2} \sin \frac{\phi}{2} |1\rangle |H\rangle, \\ |\psi_-(3)\rangle &= \sqrt{\cos \phi} |3\rangle |H\rangle - \sqrt{2} \sin \frac{\phi}{2} |-1\rangle |V\rangle \end{aligned} \quad (9)$$

During the processing, only the initial coin state  $|\psi_{\pm}\rangle$  and the coin flipping in the position  $x = 1$  for the second step  $C(1, 2, \vartheta_{1,2})$  depend on the choice of the states which need to be discriminated, which decreases the difficulty of the experimental realization.

Corresponding to the three outcomes of the measurement on the walker's position, which is realized in our experiment by the coincidence measurement of the photons in the three spatial modes  $x = 3, 1, -1$  and the trigger photons, we can discriminate the nonorthogonal states  $(|0\rangle, \alpha|0\rangle + \beta|1\rangle)$ . If the walker is measured in the position  $x = \pm 1$  we know that the coin state is  $|\psi_{\pm}\rangle$  which corresponds to the state  $|0\rangle$  (or  $\alpha|0\rangle + \beta|1\rangle$ ). If the walker

is measured in the position  $x = 3$  we know nothing about the state discrimination. The successful probability theoretically is

$$\eta = 2 \sin^2 \frac{\phi}{2} = 1 - \alpha, \quad (10)$$

which increases with  $\alpha$  decreasing. For the extreme case of  $\alpha = 0$  the measurement becomes a projective measurement and the probability of discriminating the states is 1.

The realization of unambiguous state discrimination of two equally probable single-qubit states via three-step QW are shown in Fig. 1 in detailed. The measured probability distributions for 1 to 3 steps of QW are shown in Fig. 2. The probabilities are obtained by the normalizing coincidence counts on each mode to total for the respective step. We characterize the quality of the experimental QW by its 1-norm distance [32] from the theoretical predictions according to  $d = \frac{1}{2} \sum_x |P^{\text{exp}}(x) - P^{\text{th}}(x)|$  shown in Table I. In our experiment the small distances ( $d < 0.02$ ) demonstrate strong agreement between the measured distribution and theoretic prediction after three steps.

$\alpha$	$\phi$	$ \psi(0)\rangle_c$	$\vartheta_{-1,2}$	$\vartheta_{1,2}$	$\vartheta_{0,3}$	$\eta \pm \Delta\eta$	$d \pm \Delta d$
0.707	45°	$ \psi_+\rangle$	45°	12°14'	22°30'	0.2861 ± 0.0030	0.0171 ± 0.0046
0.588	54°	$ \psi_+\rangle$	45°	15°19'	22°30'	0.4037 ± 0.0038	0.0184 ± 0.0047
0.454	63°	$ \psi_+\rangle$	45°	18°54'	22°30'	0.5362 ± 0.0045	0.0192 ± 0.0047
0.309	72°	$ \psi_+\rangle$	45°	23°18'	22°30'	0.6834 ± 0.0054	0.0127 ± 0.0046
0.156	81°	$ \psi_+\rangle$	45°	29°20'	22°30'	0.8384 ± 0.0062	0.0066 ± 0.0044
0	90°	$ \psi_+\rangle$	45°	45°	22°30'	0.9940 ± 0.0070	0.0060 ± 0.0035
0.707	45°	$ \psi_-\rangle$	45°	12°14'	22°30'	0.2875 ± 0.0031	0.0152 ± 0.0045
0.588	54°	$ \psi_-\rangle$	45°	15°19'	22°30'	0.4066 ± 0.0039	0.0156 ± 0.0047
0.454	63°	$ \psi_-\rangle$	45°	18°54'	22°30'	0.5365 ± 0.0045	0.0183 ± 0.0046
0.309	72°	$ \psi_-\rangle$	45°	23°18'	22°30'	0.6854 ± 0.0055	0.0136 ± 0.0049
0.156	81°	$ \psi_-\rangle$	45°	29°20'	22°30'	0.8394 ± 0.0063	0.0071 ± 0.0043
0	90°	$ \psi_-\rangle$	45°	45°	22°30'	0.9920 ± 0.0071	0.0080 ± 0.0036

TABLE I: The coefficients of the states to be discriminated, the initial coin states, the corresponding parameters for the HWP settings, the experimental data for the successful probabilities and 1-norm distance from the theoretical predictions. Error bars indicate the statistical uncertainty.

In our experiment, we choose different coefficient  $\alpha$  for the set of two states  $\{|0\rangle, \alpha|0\rangle + \beta|1\rangle\}$  and prepare the initial coin state to the corresponding state  $|\psi_{\pm}\rangle$  with the condition in Eq. (2) satisfied. For either of the two states, the photons undergoing through the interferometer network are measured at the modes  $x = 3$  for inclusive result and  $x = \pm 1$  for conclusive results. Two pronounced peaks for each  $\phi$  shown in the probability distribution

in Figs. 2(a) and (b) clearly prove the demonstration of the unambiguous state discrimination. With  $\alpha$  decreasing from  $1/\sqrt{2}$  to 0 the measured successful probability  $\eta$  of the discrimination of the state  $|0\rangle$  increases from  $0.2861 \pm 0.0030$  to  $0.9940 \pm 0.0070$  (from  $0.2875 \pm 0.0031$  to  $0.9920 \pm 0.0071$  to discriminate the state  $\alpha|0\rangle + \beta|1\rangle$ ) shown in Fig. 2(c). The process can be deterministic if and only if the two states are orthogonal, i.e.  $\alpha = 0$ , and

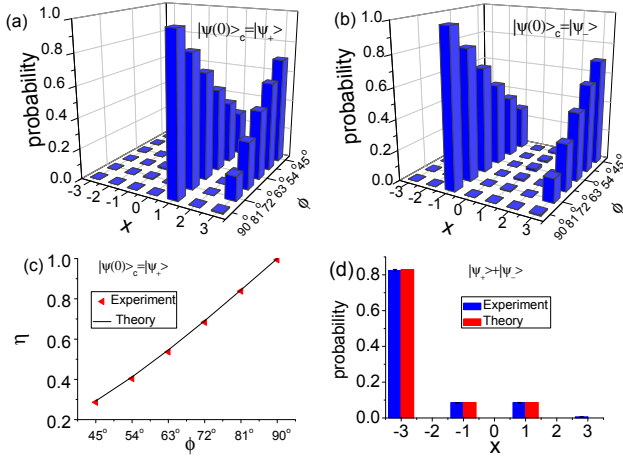


FIG. 2: (Color online.) Experimental data of the unambiguous state discrimination via a photonic QW. Measured probability distributions of the three-step QW with the position-dependent coin and initial coin state  $|\psi_+\rangle$  in (a) and  $|\psi_-\rangle$  in (b); the different coefficients  $\phi$  of  $|\psi_\pm\rangle$  for unambiguous state discrimination. (c) Measured successful probability  $\eta$  v.s. the coefficient  $\phi$  which is related to the state to be discriminated, compared to theoretical predictions. Error bars are smaller than used symbols. (d) Probability distribution of the three-step QW with the initial coin state  $|H\rangle$  which is an equally-weighted superposition of  $|\psi_\pm\rangle$  with  $\phi = 45^\circ$ . The blue and red bars show the experimental data and theoretical predictions, respectively.

the measurement becomes the projective measurement.

Taking a superposition of  $|\psi_\pm\rangle$  as an initial coin state  $|\psi(0)\rangle_c \propto a|\psi_+\rangle + b|\psi_-\rangle$  ( $a, b \in \mathbb{R}$ ) the photons undergoing through the interferometer network for the three-step QW are detected in the positions  $x = 1$  and  $x = -1$  simultaneously with probabilities  $2a^2 \sin^2 \frac{\phi}{2} / (a^2 + b^2 +$

$2ab \cos \phi)$  and  $2b^2 \sin^2 \frac{\phi}{2} / (a^2 + b^2 + 2ab \cos \phi)$  respectively for the conclusive results and in the position  $x = 3$  with the probability  $4 \cos \phi / (a^2 + b^2 + 2ab \cos \phi)$  for the inconclusive result. The ratio of the probabilities for the two conclusive results is  $a^2/b^2$ , which is also proven in our experiment. In Fig. 2(d) we show with  $a = b$  the probabilities of  $x = 1$  and  $x = -1$  are measured approximately equal, i.e.  $P(1) = 0.0854 \pm 0.0015$  and  $P(-1) = 0.0850 \pm 0.0015$ .

Therefore we have clearly demonstrated the generalized measurement scheme via a three-step QW for the first time. The unambiguous state discrimination is confirmed by direct measurement and found to be consistent with the ideal theoretical values at the level of the average distance  $d < 0.02$  and the fidelity of the coin state measured in the position  $x = \pm 1$   $F > 0.9911$ .

Now based on our experimental realization and result, we go for a QW algorithm for generation of the POVM elements corresponding to the unambiguous state discrimination. Let us consider the POVM element  $E_i$  ( $i = \pm 1$ ).

1. Start with the state  $|i\rangle |\psi_i\rangle_c$ .
2. Apply the reversed evolution operator of the three-step QW  $\tilde{U}_1^\dagger \tilde{U}_2^\dagger \tilde{U}_3^\dagger$  on  $|i\rangle |\psi_i\rangle_c$  with  $\tilde{U}_n^\dagger = C^\dagger(x, n, \vartheta_{x,n}) T^\dagger$ .
3. Take the overlap with the ancilla state (the initial state of the walker)  $|x = 0\rangle$  and obtain  $|\tilde{\psi}_i\rangle = \langle 0 | \tilde{U}_1^\dagger \tilde{U}_2^\dagger \tilde{U}_3^\dagger |i\rangle |\psi_i\rangle_c \langle \psi_i | \langle i | \tilde{U}_3 \tilde{U}_2 \tilde{U}_1 | 0 \rangle$ .
4. The POVM element for unambiguous state discrimination is then  $E_i = |\tilde{\psi}_i\rangle \langle \tilde{\psi}_i|$ .

To prove that, we have

$$\begin{aligned}
 E_+ |\psi_-\rangle &= \frac{1}{2 \cos^2 \frac{\phi}{2}} (\sin \frac{\phi}{2} |H\rangle + \cos \frac{\phi}{2} |V\rangle) (\sin \frac{\phi}{2} \langle H| + \cos \frac{\phi}{2} \langle V|) |\psi_-\rangle = 0 \\
 E_- |\psi_+\rangle &= \frac{1}{2 \cos^2 \frac{\phi}{2}} (-\sin \frac{\phi}{2} |H\rangle + \cos \frac{\phi}{2} |V\rangle) (-\sin \frac{\phi}{2} \langle H| + \cos \frac{\phi}{2} \langle V|) |\psi_+\rangle = 0.
 \end{aligned} \tag{11}$$

We experimentally prove that discrete-time QWs are capable of performing generalized measurements on a single qubit. We explicitly for the first time realize a photonic three-step QW with position-dependent coin flipping for unambiguous state discrimination of two equally probable single-qubit states with single photons undergoing through an interferometer network. Furthermore, corresponding to the unambiguous state discrimination problem the QW generation of the POVM elements is shown.

We use a very simple experiment to show the meaning of the idea on using QW to implement a generalized measurement which is shown in [26]. In a QW, the position shifts of the walker depend on the coin state being measured. That is, a QW is based on projection measurement which in the QW scheme can be extended in the evolution time. Furthermore as shown in the above experiment, the projection measurement model can naturally be extended to an arbitrary generalized measurement without the extension of the Hilbert space.

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